Remainder, Factor Theorem and Partial Fractions

Remainder and Factor Theorem

Remainder Theorem

(a) If a polynomial $P(x)$ is divided by a linear divisor $x - c$, the remainder is $P(c)$.

(b) If a polynomial $P(x)$ is divided by a linear divisor $ax + b$, the remainder is $P(-\frac{b}{a})$.

Example:

Remainder when $2x^2 + 4x - 1$ is divided by $2x + 1$ is $2\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) - 1 = -2\frac{1}{2}$

Factor Theorem

$ax + b$ is a linear factor of the polynomial $P(x)$ if and only if $P\left(-\frac{b}{a}\right) = 0$, i.e the remainder is 0.

Solving cubic equations

General Steps:

Step 1: Find the first factor by trial-and-error or using the "solve" function of the calculator. Remember to show the working using factor theorem to prove that the first factor is indeed a factor

Step 2: Use either long division or comparing coefficients to factorise the cubic equation.

Step 3: Equate the original equation to be 0 and solve the equation accordingly

Special Algebraic Identities

Sum of Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Partial Fractions

General Steps:

Step 1: Check if the fraction is improper or proper.

Step 2: If improper (highest power (degree) of the numerator is equal to or higher than the highest power of the denominator), use long division.

Example:

$$\frac{2x^3 + 2}{x^3 - 5x^2 + 2x - 13} \rightarrow Improper\ (degrees\ are\ the\ same)$$

$$\frac{2x^3 + 2}{x^2 + 2x + 13} \rightarrow Improper\ (denominator\ has\ higher\ degree)$$

$$\frac{2x^3 + 2}{4x^4 - 6x + 16} \rightarrow Proper\ (numerator\ has\ lower\ degree)$$

Step 3: Factorise the base as much as possible

Example:

$$\frac{4x}{x^2 - 2} = \frac{4x}{(x + 2)(x - 2)}$$

$$\frac{4x - 1}{x^3 - 2x^2 + x} = \frac{4x - 1}{x(x^2 - 2x + 1)} = \frac{4x - 1}{x(x - 1)^2}$$

Step 3: Express in partial fractions, according to the table below.

Case	Denominator Contains	Algebraic Fraction	Partial Fractions
1	Distinct linear factors	$\frac{px + q}{(ax + b)(cx + d)}$	$\frac{A}{ax + b} + \frac{B}{cx + d}$
2	Repeated linear factors	$\frac{px + q}{(ax + b)^2}$	$\frac{A}{ax + b} + \frac{B}{(ax + b)^2}$
3	Quadratic factor that cannot be factorised	$\frac{px + q}{(ax + b)(x^2 + c^2)}$	$\frac{A}{ax + b} + \frac{Bx + C}{x^2 + c^2}$

Step 4: Solve the partial fraction by either cover up rule or comparing coefficients or any other method.

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